

Exercice 1

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 + 16x - 6 \quad Q(x) = 2x^2 - 7x + 4 \quad R(x) = 4x^2 + 12x + 9 \quad S(x) = x^2 - 5x - 8$$

Exercice 2

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = -5x^2 + 5x - 9 \quad Q(x) = 25x^2 + 20x + 4 \quad R(x) = x^2 + 6x + 9 \quad S(x) = x^2 + 5x + 6$$

Exercice 3

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 + 4x + 2 \quad Q(x) = 25x^2 + 70x + 49 \quad R(x) = x^2 - 9x + 3 \quad S(x) = 5x^2 + x - 9$$

Exercice 4

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 + 5x + 5 \quad Q(x) = 49x^2 + 112x + 64 \quad R(x) = x^2 - 2x + 7 \quad S(x) = 4x^2 + 6x + 3$$

Exercice 5

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 16x^2 + 24x + 9 \quad Q(x) = x^2 + 7x - 8 \quad R(x) = x^2 - 8x - 8 \quad S(x) = -2x^2 + 6x - 1$$

Exercice 6

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 81x^2 - 90x + 25 \quad Q(x) = 5x^2 - 4x - 5 \quad R(x) = x^2 + 8x - 8 \quad S(x) = x^2 - 11x - 4$$

Exercice 7

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 + 2x + 9 \quad Q(x) = 4x^2 - 9x - 2 \quad R(x) = 49x^2 + 14x + 1 \quad S(x) = x^2 + 11x - 4$$

Exercice 8

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 2x^2 + 9x + 5 \quad Q(x) = 4x^2 - 12x + 9 \quad R(x) = x^2 + 11x + 5 \quad S(x) = x^2 + 10x - 5$$

Corrigé de l'exercice 1Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned}
 P(x) &= x^2 + 16x - 6 \\
 &= (x + 8)^2 - 8^2 - 6 \\
 &= (x + 8)^2 + 64 - 6
 \end{aligned}$$

$$P(x) = (x + 8)^2 + 58$$

$$\begin{aligned}
 Q(x) &= 2x^2 - 7x + 4 \\
 &= 2 \times \left(x^2 - \frac{7}{2}x + 2 \right) \\
 &= 2 \times \left(\left(x - \frac{7}{4} \right)^2 - \left(\frac{7}{4} \right)^2 + 2 \right) \\
 &= 2 \times \left(\left(x - \frac{7}{4} \right)^2 + \frac{-49}{16} + \frac{2 \times 16}{1 \times 16} \right)
 \end{aligned}$$

$$= 2 \times \left(\left(x - \frac{7}{4} \right)^2 + \frac{-49}{16} + \frac{32}{16} \right)$$

$$= 2 \times \left(\left(x - \frac{7}{4} \right)^2 + \frac{-17}{16} \right)$$

$$= 2 \times \left(x - \frac{7}{4} \right)^2 + \frac{-17 \times 2}{2 \times 8}$$

$$Q(x) = 2 \times \left(x - \frac{7}{4} \right)^2 + \frac{-17}{8}$$

$$\begin{aligned}
 S(x) &= x^2 - 5x - 8 \\
 &= \left(x - \frac{5}{2} \right)^2 - \left(\frac{5}{2} \right)^2 - 8 \\
 &= \left(x - \frac{5}{2} \right)^2 + \frac{-25}{4} - \frac{8 \times 4}{1 \times 4} \\
 &= \left(x - \frac{5}{2} \right)^2 + \frac{-25}{4} - \frac{32}{4}
 \end{aligned}$$

$$S(x) = \left(x - \frac{5}{2} \right)^2 + \frac{-57}{4}$$

$$R(x) = 4x^2 + 12x + 9$$

$$= (2x + 3)^2$$

$$= \left(2 \times \left(x + \frac{3}{2} \right) \right)^2$$

$$R(x) = 4 \times \left(x + \frac{3}{2} \right)^2$$

Corrigé de l'exercice 2Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned}
 P(x) &= -5x^2 + 5x - 9 \\
 &= -5 \times \left(x^2 - x + \frac{9}{5} \right) \\
 &= -5 \times \left(\left(x - \frac{1}{2} \right)^2 - \left(\frac{1}{2} \right)^2 + \frac{9}{5} \right) \\
 &= -5 \times \left(\left(x - \frac{1}{2} \right)^2 + \frac{-1 \times 5}{4 \times 5} + \frac{9 \times 4}{5 \times 4} \right)
 \end{aligned}$$

$$= -5 \times \left(\left(x - \frac{1}{2} \right)^2 + \frac{-5}{20} + \frac{36}{20} \right)$$

$$= -5 \times \left(\left(x - \frac{1}{2} \right)^2 + \frac{31}{20} \right)$$

$$= -5 \times \left(x - \frac{1}{2} \right)^2 + \frac{31 \times 5 \times (-1)}{5 \times 4}$$

$$P(x) = -5 \times \left(x - \frac{1}{2} \right)^2 + \frac{-31}{4}$$

$$Q(x) = 25x^2 + 20x + 4$$

$$= (5x + 2)^2$$

$$= \left(5 \times \left(x + \frac{2}{5} \right) \right)^2$$

$$Q(x) = 25 \times \left(x + \frac{2}{5} \right)^2$$

$$R(x) = x^2 + 6x + 9$$

$$= (x + 3)^2 - 3^2 + 9$$

$$= (x + 3)^2 + 9 + 9$$

$$R(x) = (x + 3)^2 + 18$$

$$S(x) = x^2 + 5x + 6$$

$$= \left(x + \frac{5}{2} \right)^2 - \left(\frac{5}{2} \right)^2 + 6$$

$$= \left(x + \frac{5}{2} \right)^2 + \frac{-25}{4} + \frac{6 \times 4}{1 \times 4}$$

$$= \left(x + \frac{5}{2} \right)^2 + \frac{-25}{4} + \frac{24}{4}$$

$$S(x) = \left(x + \frac{5}{2} \right)^2 + \frac{-1}{4}$$

Corrigé de l'exercice 3Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= x^2 + 4x + 2 \\ &= (x + 2)^2 - 2^2 + 2 \\ &= (x + 2)^2 + 4 + 2 \end{aligned}$$

$$P(x) = (x + 2)^2 + 6$$

$$\begin{aligned} R(x) &= x^2 - 9x + 3 \\ &= \left(x - \frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 + 3 \\ &= \left(x - \frac{9}{2}\right)^2 + \frac{-81}{4} + \frac{3 \times 4}{1 \times 4} \\ &= \left(x - \frac{9}{2}\right)^2 + \frac{-81}{4} + \frac{12}{4} \end{aligned}$$

$$R(x) = \left(x - \frac{9}{2}\right)^2 + \frac{-69}{4}$$

$$\begin{aligned} Q(x) &= 25x^2 + 70x + 49 \\ &= (5x + 7)^2 \\ &= \left(5 \times \left(x + \frac{7}{5}\right)\right)^2 \end{aligned}$$

$$Q(x) = 25 \times \left(x + \frac{7}{5}\right)^2$$

$$\begin{aligned} S(x) &= 5x^2 + x - 9 \\ &= 5 \times \left(x^2 + \frac{1}{5}x - \frac{9}{5}\right) \\ &= 5 \times \left(\left(x + \frac{1}{10}\right)^2 - \left(\frac{1}{10}\right)^2 + \frac{-9}{5}\right) \\ &= 5 \times \left(\left(x + \frac{1}{10}\right)^2 + \frac{-1}{100} + \frac{-9 \times 20}{5 \times 20}\right) \\ &= 5 \times \left(\left(x + \frac{1}{10}\right)^2 + \frac{-1}{100} + \frac{-180}{100}\right) \\ &= 5 \times \left(\left(x + \frac{1}{10}\right)^2 + \frac{-181}{100}\right) \\ &= 5 \times \left(x + \frac{1}{10}\right)^2 + \frac{-181 \times 5}{5 \times 20} \end{aligned}$$

$$S(x) = 5 \times \left(x + \frac{1}{10}\right)^2 + \frac{-181}{20}$$

Corrigé de l'exercice 4Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= x^2 + 5x + 5 \\ &= \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 5 \\ &= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{5 \times 4}{1 \times 4} \\ &= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{20}{4} \end{aligned}$$

$$P(x) = \left(x + \frac{5}{2}\right)^2 + \frac{-5}{4}$$

$$\begin{aligned} Q(x) &= 49x^2 + 112x + 64 \\ &= (7x + 8)^2 \\ &= \left(7 \times \left(x + \frac{8}{7}\right)\right)^2 \end{aligned}$$

$$Q(x) = 49 \times \left(x + \frac{8}{7}\right)^2$$

$$R(x) = x^2 - 2x + 7$$

$$= (x - 1)^2 - 1^2 + 7$$

$$= (x - 1)^2 + 1 + 7$$

$$R(x) = (x - 1)^2 + 8$$

$$\begin{aligned} S(x) &= 4x^2 + 6x + 3 \\ &= 4 \times \left(x^2 + \frac{3}{2}x + \frac{3}{4}\right) \\ &= 4 \times \left(\left(x + \frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 + \frac{3}{4}\right) \\ &= 4 \times \left(\left(x + \frac{3}{4}\right)^2 + \frac{-9}{16} + \frac{3 \times 4}{4 \times 4}\right) \end{aligned}$$

$$= 4 \times \left(\left(x + \frac{3}{4}\right)^2 + \frac{-9}{16} + \frac{12}{16}\right)$$

$$= 4 \times \left(\left(x + \frac{3}{4}\right)^2 + \frac{3}{16}\right)$$

$$= 4 \times \left(x + \frac{3}{4}\right)^2 + \frac{3 \times 4}{4 \times 4}$$

$$S(x) = 4 \times \left(x + \frac{3}{4}\right)^2 + \frac{3}{4}$$

Corrigé de l'exercice 5Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 16x^2 + 24x + 9$$

$$= (4x + 3)^2$$

$$= \left(4 \times \left(x + \frac{3}{4}\right)\right)^2$$

$$P(x) = 16 \times \left(x + \frac{3}{4}\right)^2$$

$$Q(x) = x^2 + 7x - 8$$

$$= \left(x + \frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 - 8$$

$$= \left(x + \frac{7}{2}\right)^2 + \frac{-49}{4} - \frac{8 \times 4}{1 \times 4}$$

$$= \left(x + \frac{7}{2}\right)^2 + \frac{-49}{4} - \frac{32}{4}$$

$$Q(x) = \left(x + \frac{7}{2}\right)^2 + \frac{-81}{4}$$

$$R(x) = x^2 - 8x - 8$$

$$= (x - 4)^2 - 4^2 - 8$$

$$= (x - 4)^2 + 16 - 8$$

$$R(x) = (x - 4)^2 + 8$$

$$S(x) = -2x^2 + 6x - 1$$

$$= -2 \times \left(x^2 - 3x + \frac{1}{2}\right)$$

$$= -2 \times \left(\left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{1}{2}\right)$$

$$= -2 \times \left(\left(x - \frac{3}{2}\right)^2 + \frac{-9}{4} + \frac{1 \times 2}{2 \times 2}\right)$$

$$= -2 \times \left(\left(x - \frac{3}{2}\right)^2 + \frac{-9}{4} + \frac{2}{4}\right)$$

$$= -2 \times \left(\left(x - \frac{3}{2}\right)^2 + \frac{-7}{4}\right)$$

$$= -2 \times \left(x - \frac{3}{2}\right)^2 + \frac{-7 \times 2 \times (-1)}{2 \times 2}$$

$$S(x) = -2 \times \left(x - \frac{3}{2}\right)^2 + \frac{7}{2}$$

Corrigé de l'exercice 6Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 81x^2 - 90x + 25$$

$$= (9x - 5)^2$$

$$= \left(9 \times \left(x - \frac{5}{9}\right)\right)^2$$

$$P(x) = 81 \times \left(x - \frac{5}{9}\right)^2$$

$$Q(x) = 5x^2 - 4x - 5$$

$$= 5 \times \left(x^2 - \frac{4}{5}x - 1\right)$$

$$= 5 \times \left(\left(x - \frac{2}{5}\right)^2 - \left(\frac{2}{5}\right)^2 - 1\right)$$

$$= 5 \times \left(\left(x - \frac{2}{5}\right)^2 + \frac{-4}{25} - \frac{1 \times 25}{1 \times 25}\right)$$

$$= 5 \times \left(\left(x - \frac{2}{5}\right)^2 + \frac{-4}{25} - \frac{25}{25}\right)$$

$$= 5 \times \left(\left(x - \frac{2}{5}\right)^2 + \frac{-29}{25}\right)$$

$$= 5 \times \left(x - \frac{2}{5}\right)^2 + \frac{-29 \times 5}{5 \times 5}$$

$$Q(x) = 5 \times \left(x - \frac{2}{5}\right)^2 + \frac{-29}{5}$$

$$R(x) = x^2 + 8x - 8$$

$$= (x + 4)^2 - 4^2 - 8$$

$$= (x + 4)^2 + 16 - 8$$

$$R(x) = (x + 4)^2 + 8$$

$$S(x) = x^2 - 11x - 4$$

$$= \left(x - \frac{11}{2}\right)^2 - \left(\frac{11}{2}\right)^2 - 4$$

$$= \left(x - \frac{11}{2}\right)^2 + \frac{-121}{4} - \frac{4 \times 4}{1 \times 4}$$

$$= \left(x - \frac{11}{2}\right)^2 + \frac{-121}{4} - \frac{16}{4}$$

$$S(x) = \left(x - \frac{11}{2}\right)^2 + \frac{-137}{4}$$

Corrigé de l'exercice 7Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= x^2 + 2x + 9 \\ &= (x+1)^2 - 1^2 + 9 \\ &= (x+1)^2 + 1 + 9 \end{aligned}$$

$$P(x) = (x+1)^2 + 10$$

$$\begin{aligned} Q(x) &= 4x^2 - 9x - 2 \\ &= 4 \times \left(x^2 - \frac{9}{4}x - \frac{1}{2} \right) \\ &= 4 \times \left(\left(x - \frac{9}{8} \right)^2 - \left(\frac{9}{8} \right)^2 + \frac{-1}{2} \right) \\ &= 4 \times \left(\left(x - \frac{9}{8} \right)^2 + \frac{-81}{64} + \frac{-1 \times 32}{2 \times 32} \right) \end{aligned}$$

$$= 4 \times \left(\left(x - \frac{9}{8} \right)^2 + \frac{-81}{64} + \frac{-32}{64} \right)$$

$$= 4 \times \left(\left(x - \frac{9}{8} \right)^2 + \frac{-113}{64} \right)$$

$$= 4 \times \left(x - \frac{9}{8} \right)^2 + \frac{-113 \times 4}{4 \times 16}$$

$$Q(x) = 4 \times \left(x - \frac{9}{8} \right)^2 + \frac{-113}{16}$$

$$\begin{aligned} S(x) &= x^2 + 11x - 4 \\ &= \left(x + \frac{11}{2} \right)^2 - \left(\frac{11}{2} \right)^2 - 4 \\ &= \left(x + \frac{11}{2} \right)^2 + \frac{-121}{4} - \frac{4 \times 4}{1 \times 4} \\ &= \left(x + \frac{11}{2} \right)^2 + \frac{-121}{4} - \frac{16}{4} \end{aligned}$$

$$S(x) = \left(x + \frac{11}{2} \right)^2 + \frac{-137}{4}$$

$$R(x) = 49x^2 + 14x + 1$$

$$= (7x+1)^2$$

$$= \left(7 \times \left(x + \frac{1}{7} \right) \right)^2$$

$$R(x) = 49 \times \left(x + \frac{1}{7} \right)^2$$

Corrigé de l'exercice 8Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= 2x^2 + 9x + 5 \\ &= 2 \times \left(x^2 + \frac{9}{2}x + \frac{5}{2} \right) \\ &= 2 \times \left(\left(x + \frac{9}{4} \right)^2 - \left(\frac{9}{4} \right)^2 + \frac{5}{2} \right) \\ &= 2 \times \left(\left(x + \frac{9}{4} \right)^2 + \frac{-81}{16} + \frac{5 \times 8}{2 \times 8} \right) \end{aligned}$$

$$= 2 \times \left(\left(x + \frac{9}{4} \right)^2 + \frac{-81}{16} + \frac{40}{16} \right)$$

$$= 2 \times \left(\left(x + \frac{9}{4} \right)^2 + \frac{-41}{16} \right)$$

$$= 2 \times \left(x + \frac{9}{4} \right)^2 + \frac{-41 \times 2}{2 \times 8}$$

$$P(x) = 2 \times \left(x + \frac{9}{4} \right)^2 + \frac{-41}{8}$$

$$\begin{aligned} Q(x) &= 4x^2 - 12x + 9 \\ &= (2x-3)^2 \\ &= \left(2 \times \left(x - \frac{3}{2} \right) \right)^2 \end{aligned}$$

$$Q(x) = 4 \times \left(x - \frac{3}{2} \right)^2$$

$$S(x) = x^2 + 10x - 5$$

$$= (x+5)^2 - 5^2 - 5$$

$$= (x+5)^2 + 25 - 5$$

$$S(x) = (x+5)^2 + 20$$

$$\begin{aligned} R(x) &= x^2 + 11x + 5 \\ &= \left(x + \frac{11}{2} \right)^2 - \left(\frac{11}{2} \right)^2 + 5 \\ &= \left(x + \frac{11}{2} \right)^2 + \frac{-121}{4} + \frac{5 \times 4}{1 \times 4} \\ &= \left(x + \frac{11}{2} \right)^2 + \frac{-121}{4} + \frac{20}{4} \end{aligned}$$

$$R(x) = \left(x + \frac{11}{2} \right)^2 + \frac{-101}{4}$$